# Spatially Contiguous Land Management: a sealed bid auction format 

Patrice LOISEL ${ }^{1}$


#### Abstract

Ecosystem services are deteriorating. It is essential to develop economic instruments that promote the production of ecosystem services. Conservation agencies use, among other things, payment systems for ecosystem services that remunerate private landowners to adopt proenvironmental practices on their spatially contiguous lands. Iterative or sealed bidding procedures are well suited to provide efficient incentive systems. Experiments have shown the superiority of iterative auctions. In order to better understand the processes implemented, we propose here to analyze the strategies of the landowners in the case of sealed bids auction format.


Keywords: Conservation auction ; ecosystem services ; coordination

JEL Codes: D44, Q15, Q24, Q57

## 1 Introduction

Ecosystem services are deteriorating, developing adapted economic instruments is a new challenge at the global level. In the majority of cases, threatened ecosystems are located on private land (Rolfe et al., 2009). It is therefore essential to look for measures that promote the production of ecosystem services by private landowners. Conservation agencies use, among other

[^0]things, payment systems for ecosystem services that remunerate private landowners to adopt pro-environmental practices on their land. The production of ecosystem services very often requires coordination of spatially contiguous land management. As a result, auction procedures seem well suited to provide efficient incentive systems. Different procedures are possible ranging from sealed bidding auction format to iterative bidding auction format.

In experimental studies Rolfe et al. (2005) and Reeson et al. (2008) considered problems of spatial agglomeration. They evaluated bid performance for creating connected landscapes. The experiments of Rolfe et al. have been implemented with real landowners using both sealed bidding and iterative auctions. For iterative auction formats, landowners were acquiring information each round, while for the sealed sealed bid format, landowners may communicate. Experiments have shown that the spatial patterns obtained are less expensive in the case of iterative auctions. Based on a scoring rule, Banerjee et al. (2015) consider a Conservation Auction model. They study an iterative descending-price auction, that explicitly includes the spatial objective into the selection criterion in the presence of a limited fixed budget.

We propose here to analyze in detail the strategies of the landowners in the case of a specific sealed bids auction. In order to clarify the impact of communication, we make the assumption in our model that landowner do not communicate with each other. We deduce behavior of the bidder strategies with respect to the landowner type.

## 2 The auction design

We consider a set of $N$ landowners. Each landowner participates by submitting an amount of financial compensation to the auction. In exchange for this financial compensation, each owner undertakes to carry out preservation practices on his property.

We assume that each participant $i$ has a private $\operatorname{cost} c_{i}$ and that this private value is drawn in an $F$ distribution. Each participant $i$ submits an amount $b_{i}$. $x_{i}$ indicates the winning or losing character of the participant $i: x_{i}=1$ if $i$ wins, $x_{i}=0$ if $i$ loses. Each winning participant in the auction receives the amount of their bid.

From the point of view of the environmental agency, benefits are expected for each individual participant but also and especially when two contiguous participants adhere to the conservation process. We consider linear configuration. All except the participants located at the extremities of the stream, interior participants have two neighbours. This translates into the environmental value function:

$$
V\left(\left(x_{i}\right)_{i}\right)=m \sum_{i=1}^{N} x_{i}+2 w \sum_{i=1}^{N-1} x_{i} x_{i+1}
$$

where $m$ is the individual benefit parameter and $2 w$ the joint benefit parameter for two neighbours.

The cost corresponding to the submissions is given by: $\sum_{i=1}^{N} b_{i} x_{i}$. Two alternatives are available: the cost is limited to a given level or is integrated in the gain function:

$$
G\left(\left(x_{i}\right)_{i},\left(b_{i}\right)_{i}\right)=m \sum_{i=1}^{N} x_{i}+2 w \sum_{i=1}^{N-1} x_{i} x_{i+1}-\sum_{i=1}^{N} b_{i} x_{i}
$$

We consider the second case.

## Continuous Distribution

If we assume that distribution $F$ is continuously differentiable, based on the first-order optimality condition, in case of uniform distribution $F$ it is possible to deduce a continuous
strategy (see Appendix):

$$
B_{l}(c)= \begin{cases}m & \text { if } c<c_{*}  \tag{1}\\ B_{0}(c) & \text { if } c_{*}<c \leq \min \left(\phi\left(c_{*}\right), \bar{c}\right)\end{cases}
$$

with $B_{0}$ and $\phi$ linear in $c$, but also discontinuous strategies parametrized by $x_{*}$ :

$$
B_{x_{*}}(c)= \begin{cases}b_{1} & \text { if } c<x_{*}  \tag{2}\\ b_{2} & \text { if } c>x_{*}\end{cases}
$$

## An iterative procedure

To obtain the bidders' strategy, an alternative procedure based on the definition of Nash equilibrium can also be used. It consists in constructing successive approximations of the bidder strategy $B_{1}, . ., B_{n}$ based on maximizing the expected gain of the participant $E\left[c_{i}, b \mid B_{n-1}().\right]$ assuming that the other participants use the bidder strategy $B_{n-1}$ :

$$
E\left[c_{i}, B_{n}\left(c_{i}\right)\right]=\max _{b} E\left[c_{i}, b \mid B_{n-1}(.)\right]
$$

Using this numerical procedure, starting with $B_{1}=B_{l}$, depending on the choosen estimation of the expectation, the method converges to one of the discrete solutions $B_{x_{*}}$. This confirms the existence of multiple equilibrium.

## Discrete Distribution

In order to remove the ambiguities relating to the multiplicity of equilibrium we consider the case of discrete distributions for which we hope to be able to study in a more precise way. Assume that the private cost take $K$ different values in ascending order $c^{1}<c^{2}<. .<c^{K}$. The
discrete density of the private cost is: $f(c)=\sum_{k=1}^{K} \theta_{k} \delta_{c^{k}}$ with $\sum_{k=1}^{K} \theta_{k}=1$. For $K=2$ we obtain:

Proposition 2.1 Assuming a discrete density of the private cost for two participants ( $N=2$ ) with two different type values ( $K=2$ ) and the environmental agency maximizes a nonegative gain $G\left(x_{i}, b_{i}\right)$ then:
(i) if $c^{2} \geq m+w,\left(1-\theta_{1}\right) c^{1}+\left(1+\theta_{1}\right) c^{2} \leq 2 m+2 w$ and a small imprecision on joint benefit parameter $w$ the bidder strategies are given by:

$$
\begin{aligned}
& b^{1}=2 m+2 w-c^{2} \\
& b^{2}=c^{2}
\end{aligned}
$$

(ii) if $c^{2}<m+w$ and $\left(1+\theta_{1}\right) c^{2}-\theta_{1} c^{1} \leq m+w$ then the bidder strategies are given by:

$$
b^{1}=b^{2}=m+w .
$$

Proof: The gain net of costs to be maximize by the environmental agency is:

$$
\max _{x_{1}, x_{2} \in\{0,1\}} G\left(x_{1}, x_{2}, b_{1}, b_{2}\right)=m\left(x_{1}+x_{2}\right)+2 w x_{1} x_{2}-\left(b_{1} x_{1}+b_{2} x_{2}\right)
$$

Each agent maximizes his expected gain:

$$
E\left(c_{i}, b\right)=\left(b-c_{i}\right) \operatorname{Pr}\left[x_{i}=1\right]
$$

We consider a Nash equilibrium, each agent follows the bid strategy $B($.$) hence: \max _{b} E\left(c_{i}, b\right)=$ $E\left(c_{i}, B\left(c_{i}\right)\right)$. Denote the optimal submission $b^{1}, b^{2}$ for the participant types. We successively consider the two bidder types $c^{1}$ and $c^{2}$.
(i) If $c^{2}>m+w, b^{2} \geq c^{2}$, for bidder with higher cost $c^{2}$, he cannot win alone, hence
$b \leq 2 m+2 w-b^{1}$, his expected gain is increasing with $b$, hence $b^{2}=2 m+2 w-b^{1}$, so $b^{1}<m+w<b^{2}$.

For bidder with lower value $c^{1}$, he is sure to win with $b \leq b^{1}$ and has a probability of winning $\theta_{1}$ with $b^{1}<b \leq b^{2}$, hence he respectively may bid $b^{1}$ and $b^{2}$, hence for a Nash equilibrium:

$$
E_{1}\left(c^{1}, b^{1}\right)=b^{1}-c^{1} \geq \theta_{1}\left(b^{2}-c^{1}\right)=E_{1}\left(c^{1}, b^{2}\right)
$$

Hence at optimum $b^{1}$ will be the highest value such that: $b^{1}-c^{1} \geq \theta_{1}\left(b^{2}-c^{1}\right), b^{1}+b^{2}=2 m+2 w$ then $b^{1}=2 m+2 w-b^{2}$ and we deduce $c^{1}\left(1-\theta_{1}\right)+b^{2}\left(1+\theta_{1}\right) \leq 2 m+2 w$ and:

$$
c^{2} \leq b^{2} \leq \frac{2}{1+\theta_{1}}(m+w)-\frac{1-\theta_{1}}{1+\theta_{1}} c^{1}
$$

Reversely optimal $b^{2}$ will be such that:

$$
E_{2}\left(c^{2}, b^{2}\right)=\theta_{1}\left(b^{2}-c^{2}\right) \geq b^{1}-c^{2}=E_{2}\left(c^{2}, b^{1}\right)
$$

so, from $b^{2}=2 m+2 w-b^{1}$ we deduce: $\left(1+\theta_{1}\right) b^{2} \geq 2(m+w)-\left(1-\theta_{1}\right) c^{2}$.
Then $\frac{2}{1+\theta_{1}}(m+w)-\frac{1-\theta_{1}}{1+\theta_{1}} c^{2} \leq b^{2} \leq \frac{2}{1+\theta_{1}}(m+w)-\frac{1-\theta_{1}}{1+\theta_{1}} c^{1}$. Once again we get an infinity of solutions.

To remove this ambiguity we consider we relax hypothesis that $w$ is perfectly known by the participants. We assume that $w=w_{1}+\rho\left(w_{2}-w_{1}\right)$ and $\rho$ is drawn from the cumulative distribution $G$ with support $[0,1]$ and $w_{2}-w_{1}$ arbitrarily small. In this case, the probability that the principal gain is positive is equal to:
$\operatorname{Pr}\left[b^{1}+b^{2} \leq 2 m+2 w_{1}+2 \rho\left(w_{2}-w_{1}\right)\right]=\operatorname{Pr}\left[\rho \geq \frac{b^{1}+b^{2}-2 m-2 w_{1}}{2\left(w_{2}-w_{1}\right)}\right]=1-G\left(\frac{b^{1}+b^{2}-2 m-2 w_{1}}{2\left(w_{2}-w_{1}\right)}\right)$
the expected gain for participant of type 2 is given by:

$$
E_{2}\left(b^{2}, c^{2}\right)=\theta_{1}\left(b^{2}-c^{2}\right)\left(1-G\left(\frac{b^{1}+b^{2}-2 m-2 w_{1}}{2\left(w_{2}-w_{1}\right)}\right) \geq b^{1}-c^{2}\right.
$$

From $b^{1}<c^{2}$, the inequality is always valid. The maximisation with respect to $b^{2}$ gives:

$$
2\left(w_{2}-w_{1}\right)\left(1-G\left(\frac{b^{1}+b^{2}-2 m-2 w_{1}}{2\left(w_{2}-w_{1}\right)}\right)\right)=\left(b^{2}-c^{2}\right) G^{\prime}\left(\frac{b^{1}+b^{2}-2 m-2 w_{1}}{2\left(w_{2}-w_{1}\right)}\right)
$$

hence assuming $\inf _{x} G^{\prime}(x)>0, b^{2}$ tends to $c^{2}$ as $w_{2}-w_{1}$ tends to zero.
(ii) If $c_{2} \leq m+w$, two cases are available $2 b^{1}=2 b^{2}=2 m+2 w$ or $b^{1}+b^{2}=2 m+2 w$ with $b^{1}<b^{2}$. In the first case, the two participants always win, the gain are respectively: $m+w-c^{1}, m+w-c^{2}$. Considering the second case multiple strategies are available but we successively deduce:

$$
\begin{aligned}
\theta_{1}\left(b^{2}-c^{1}\right) & \leq b^{1}-c^{1}=2 m+2 w-b^{2}-c^{1} \\
\theta_{1}\left(b^{2}-c^{2}\right) & \leq 2 m+2 w-b^{2}-\left(1-\theta_{1}\right) c^{1}-\theta_{1} c^{2} \\
\left(1+\theta_{1}\right)\left(b^{2}-c^{2}\right) & \leq 2 m+2 w-\left(1-\theta_{1}\right) c^{1}-\left(1+\theta_{1}\right) c^{2} \\
\theta_{1}\left(b^{2}-c^{2}\right) & \leq \frac{2 \theta_{1}}{1+\theta_{1}}(m+w)-\theta_{1} \frac{1-\theta_{1}}{1+\theta_{1}} c^{1}-\theta_{1} c^{2}
\end{aligned}
$$

Hence the gain difference between the second and the first case satisfies:
$\theta_{1}\left(b^{2}-c^{2}\right)-\left(m+w-c^{2}\right) \leq-\frac{1-\theta_{1}}{1+\theta_{1}}\left(m+w+\theta_{1} c^{1}-\left(1+\theta_{1}\right) c^{2}\right)<0$
hence gain is larger in the first case for the two types of participants.

If $c^{2}<m+w$ and $\left(1+\theta_{1}\right) c^{2}-\theta_{1} c^{1}>m+w$, there is multiple strategies.

Participant gain can be lower for participant of higher type.

Let us return to the problem with uniform distribution $F$ and consider the iterative procedure. Starting from the same starting point $B_{l}$ but this time taking into account an imprecision on $w$ the method converges to $B_{l}$. We can therefore conjecture that $B_{l}$ is the solution.

## 3 Discussion and conclusion

We show that it is possible to obtain the bidder strategies. In some cases, imprecision on joint benefit parameter $w$ must be considered. We find that the landowner with the largest cost may have a lower gain.

For $N>2$, bidder strategies are attainable by means of further calculations. It would be possible to test the confirmation of the previous result in a more general framework. In conclusion, we showed that it is possible to obtain the bidder strategies by considering inequality conditions.

## References

Banerjee, S., Kwasnica, A.M. and Shortle, J.S. 2015. Information and auction performance: A laboratory study of conservation auctions for spatially contiguous land management. Environmental and Resource Economics 61: 409.

Reeson, A.F., Rodriguez, L., Whitten, S.M., Williams, K.J., Nolles, K., Windle, J., Rolfe, J. 2008. Applying competitive tenders for the provision of ecosystem services at the landscape
scale: Paper. Applying Competitive Tenders for the Provision of Ecosystem Services at the Landscape Scale. Paper accepted for 10th International BIOECON Conference, Cambridge.

Rolfe, J., and J. Windle. 2006. Using field experiments to explore the use of multiple bidding rounds in conservation auctions. Canadian Journal of Agricultural Economics.

Rolfe, J., J. Windle, and J. McCosker. 2009. Testing and implementing the use of multiple bidding rounds in conservation auctions: A case study application. Canadian Journal of Agricultural Economics/Revue Canadienne d'Agroeconomie 57 (3): 287-303.

## Appendix

We assume that the principal has a non negative gain. Each agent has a private value with distribution $F$ of finite support and maximises his expected gain:

$$
E\left(c_{1}, b\right)=\left(b-c_{1}\right) \operatorname{Pr}\left[x_{1}=1\right]
$$

We consider a Nash equilibrium, each agent follows the bid strategy $B($.$) hence: \max _{b} E\left(c_{1}, b\right)=$ $E\left(c_{1}, B\left(c_{1}\right)\right)$

Let $N=2$, the program of the principal is to maximize his gain:

$$
\max \left(m-b_{1}, m-b_{2}, 2 m+2 w-b_{1}-b_{2}\right)
$$

If $b_{1}>m+2 w$ then $2 m+2 w-b_{1}-b_{2}<m-b_{2}$ so $x_{1}=0$. If $b_{1}<m+2 w$ then $m-b_{2}<$ $2 m+2 w-b_{1}-b_{2}$ so the maximum reached for the principal is $\max \left(m-b_{1}, 2 m+2 w-b_{1}-b_{2}\right)$. We deduce that if the principal wants to insure a gain greater than $G_{0}$, the bid $b$ of the agent is
given by the maximum:

$$
\max \left(\left(m-c_{1}\right) I_{m-c_{1} \geq 0},\left(b-c_{1}\right) \operatorname{Pr}\left[2 m+2 w-b-B\left(u_{2}\right) \geq 0\right]\right)
$$

with $\operatorname{Pr}\left[2 m+2 w-b-B\left(u_{2}\right) \geq 0\right]=F\left(B^{-1}(2 m+2 w-b)\right)$. We first consider the problem of the right program which maximises $\left.\left(b-c_{1}\right) \operatorname{Pr}\left[2 m+2 w-b-B_{0}\left(u_{2}\right) \geq 0\right]\right)$ with bid strategy $B_{0}$. The corresponding first-order condition is given by:

$$
F\left(B_{0}^{-1}\left(2 m+2 w-B_{0}(c)\right)\right)=\left(B_{0}(c)-c\right) \frac{F^{\prime}}{B_{0}^{\prime}}\left(B_{0}^{-1}\left(2 m+2 w-B_{0}(c)\right)\right)
$$

Let the function $\phi$ defined by: $B_{0}(\phi(c))=2 m+2 w-B_{0}(c)$, we deduce:

$$
\begin{align*}
& B_{0}^{\prime}(\phi(c)) F(\phi(c))=\left(B_{0}(c)-c\right) F^{\prime}(\phi(c))  \tag{3}\\
& B_{0}(c)+B_{0}(\phi(c))=2 m+2 w \tag{4}
\end{align*}
$$

Assume $F$ uniform distribution, $F(x)=x$ then, the system has a linear solution: $B_{0}(c)=$ $\frac{2}{3}(m+w)+\frac{c}{2}$ with $\phi(c)=\frac{4}{3}(m+w)-c$. Due to limited value for bid strategy we deduce that the maximal private value which permits the agent to bid satisfies: $\frac{1}{3}(2 m+2 w)+\frac{v}{2} \leq m+2 w$ i.e. $c \leq \bar{c}$ with $\bar{c}=\frac{2 m+8 w}{3}$. Let $c_{*}$ the private value at which the agent changes of strategy:

$$
\begin{aligned}
m-c_{*} & =\left(B_{0}\left(c_{*}\right)-c_{*}\right) \phi\left(c_{*}\right)=\left(\frac{2}{3}(m+w)-\frac{c_{*}}{2}\right)\left(\frac{4}{3}(m+w)-c_{*}\right) \\
2\left(m-c_{*}\right) & =\left(\frac{4}{3}(m+w)-c_{*}\right)^{2}
\end{aligned}
$$

Hence $B(c)=m$ for $c<c_{*}$. Moreover $B_{0}\left(\phi\left(c_{*}\right)\right)=2 m+2 w-B_{0}\left(c_{*}\right)=\phi\left(c_{*}\right)$. As
$B_{0}^{\prime}(c)=\frac{1}{2}$, so it is impossible that $B_{0}(c) \geq c$ for $c>\phi\left(c_{*}\right)$. Hence:

$$
B_{l}(c)= \begin{cases}m & \text { if } c<c_{*}  \tag{5}\\ B_{0}(c) & \text { if } c_{*}<c \leq \min \left(\phi\left(c_{*}\right), \bar{c}\right)\end{cases}
$$

But we have not shown the uniqueness of the solution. So other strategies are available, notably strategies provided by step functions. And this is indeed the case, if we consider a $B$ function defined by:

$$
B(c)= \begin{cases}b_{1} & \text { if } c<x_{*}  \tag{6}\\ b_{2} & \text { if } c>x_{*}\end{cases}
$$

with $b_{1} \geq x_{*}$.
The optimality conditions are the following:

$$
\begin{cases}b_{1}-c \geq x_{*}\left(b_{2}-c\right) & \text { if } c<x_{*}  \tag{7}\\ x_{*}\left(b_{2}-c\right) \geq b_{1}-c & \text { if } c \geq x_{*}\end{cases}
$$

with $b_{1}+b_{2}=2 m+2 w$. The inequalities are checked if and only if:

$$
\begin{align*}
b_{1}-x_{*} & \geq x_{*}\left(b_{2}-x_{*}\right)  \tag{8}\\
x_{*}\left(b_{2}-x_{*}\right) & \geq b_{1}-x_{*} \tag{9}
\end{align*}
$$

Hence $b_{1}=b_{2} x_{*}+x_{*}\left(1-x_{*}\right)$, from $b_{1}+b_{2}=2 m+2 w$ we deduce $\left(b_{1}-x_{*}\right)\left(1-x_{*}\right)=$ $2(m+w) x_{*}$. So it exists an infinity of function $B$ satisfying the optimality conditions.


[^0]:    ${ }^{1}$ MISTEA, INRA, Montpellier SupAgro, Univ Montpellier, Montpellier, France

